Construction and Selection of Two Sided Complete Chain Sampling Plans - CCHSP (0, 1) Indexed Through AOQL

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Abstract:
In modern quality control practice, the professionals insist that if any defect occurs in a sample then very strict quality control measures should be adopted in order to avoid rejection. A novel algorithm for Chain Sampling Inspection called, Two sided - Complete Chain Sampling Plans CChSP(0,1) with outgoing quality limit as indexing is developed and presented in this paper. The complete Chain sampling plan gives more protection to the consumer while giving more pressure on the producer. Comparison between Ordinary Chain and Complete Chain sampling plans shows better discrimination in sentencing the good lot against the bad one. These plans provide small sample sizes by ensuring protection for the producers and at the same time assuring the consumer with the better quality level after the inspection. Tables are constructed for two sided complete chain sampling plan indexed through AOQL and illustration is also given for easy selection of the plan.

Keywords: Chain sampling, Two Sided Complete Chain sampling, OC(operating characteristic) function, AOQL.

Introduction
The Chain Sampling Plans developed by many authors are only of partial chaining. That is, it will chain only the past lots to decide about the current lot or it may defer the decision until few sample results are obtained. But the literature is not adequate in development plans satisfying the results of past, current and future lots. In modern Industries the future samples are also possible which can be utilized in sentencing the current lot. There are possibilities in many production industries for recording past, current and future sample results. Hence in this paper two sided complete chain sampling plan is being developed by considering the results of past as well as future lots if the current sample does not lead to acceptance.

In Literature, Several Sampling Plans are available to accept or reject a lot. Dodge (1955) has introduced the ChSP (0,1) plans for small sampling and costly situations. Dodge et.al,(1966) have given new type of Chain sampling inspection plans. Clark., (1955) has developed OC curves for Chain sampling plans. Frishman and Fred (1960) have developed an extended Chain sampling plan. Dodge and Stephens (1974) have contributed towards evaluation of OC of Chain sampling plans through Markov Chains. Soundararajan., (1971& 1978) has given a procedure and tables for construction and selection of Chain sampling plans. Kuralmani and Govindaraj (1993) have contributed towards the selection of Conditional sampling plans for given AQL and LQL. Raju., (1992) has developed OC functions for certain Conditional sampling plans. Suresh K.K. and Devaarul S (2002) have developed Mixed Sampling Plans with Chain Sampling as attribute plan.
In the present competitive scenario, many quality control practitioners insist that if any defect occurs in a sample then not only the preceding ‘i’ samples but the succeeding ‘j’ samples should also be considered, for the decision of the current lot. Hence an attempt has been made to develop Complete Chain Sampling Plans by the authors. The CChsp(0,1) plans are developed by assuming that there are possibilities in many production industries for past, current and future samples.

**Operating procedure of Cchsp (0,1)**

The algorithm for sentencing a lot or batch was developed by Devaarul and Edna (2011) as follows:

(i) For each lot, select a sample of n units and test each unit for conformance to the specified attribute standard.

(ii) Accept the current lot if d (the observed number of defectives) is zero in the sample of n units and reject the lot if d > 1. If d = 1, go to next step.

(iii) Now accept the current lot if d = 1 and if no defectives are found in the immediately preceding ‘i’ samples and succeeding ‘j’ samples from the same steady state process.

**Measures of Sampling Plans**

One of the Measure of Sampling Plan is Operating Characteristics Curve which reveals the power of discrimination if bad quality of product prevails in the production process. Devaarul and Edna (2010) have derived the new \( OC_n, ASN \) of CChsp(0,1) and are given below for the reference.

**3.1 Operating Characteristic Curve**

\[
(i) \quad P_a(p) = P_0 + P_0^i P_0^j \quad \text{if } i \neq j
\]

\[
(ii) \quad P_a(p) = P_0 + P_0^{2i} P_1 \quad \text{if } i=j
\]

\[
= P_0[1 + P_0^{2i-1} P_1]
\]

**Average Sample Number**

\[
ASN = nP_o + inP_o^i nP_1 jnP_o^j
\]

\[
ASN = nP_0 + i^2 n^3 P_o^{2i} \quad \text{if } i=j
\]

\[
ASN = n \quad \text{if current lot has Zero defective}
\]

\[
= i^2 n^3 P_o^{2i}, \quad \text{if the current lot has more than Zero defective.}
\]

**Theorem :** AOQL limit of CChSP(0,1) is \( p_m \) and is a function of ‘i’ alone.
The OC function of CChSP(0,1) is defined as

\[ P_a(p) = P_0 + P_0 P_1 P_j \] if \( i \neq j \)

\[ \text{AOQ} = p \cdot P_a(p) \] (1)

\[ \text{AOQ} = p e^{-np} + np \cdot e^{-np(1+2i)} \]

\[ \text{AOQ} = pe^{-np} + np^2 \cdot e^{-np(1+2i)} \]

Maximizing (1) with respect to the incoming quality (p), we get

\[
\frac{d\text{AOQ}}{dp} = -np e^{-np} + e^{-np} + n \left[ p^2 (-n(1+2i)) e^{-np(1+2i)} + 2pne^{-np(1+2i)} \right]
\]

When \( \frac{d^2\text{AOQ}}{d^2p} = 0 \)

\[-np e^{-np} + e^{-np} - n^2 p^2 (1+2i) e^{-np(1+2i)} + 2pne^{-np(1+2i)} = 0\]

Divide by \( e^{-np} \) on both sides

\[-np + 1 - n^2 p^2 (1+2i) e^{-np2i} + 2pne^{-np2i} = 0\]

\[1 - np - e^{-np2i} [n^2 p^2 (1+2i) - 2pn] = 0\]

\[e^{-np2i} [n^2 p^2 (1+2i) - 2pn] = (1 - np)\]

\[e^{-np2i} = \frac{(1 - np)}{[n^2 p^2 (1+2i) - 2pn]}\] (4)

(4) is a function of ‘i’ and np

Using (4), tables are constructed for easy selection of the plan and is given in table 1. The table can be extended.
Table 1: Values of the sample size $n$ for the known $i$ and AOQL.

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Table (1) cont...

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Illustration:
Determine the Cchsp(0,1) at AOQL= .38%. and chaining index i=1.
For the above quality requirement, from the table 1, n= 300, i=1 and AOQ=0.0038 then 
n AOQL =1.14.
Step 1 : Take a random sample of size n=300 from a lot.
Step 2 : Count the number of defective ‘d’
Step 3 : If the number of defectives d=0 , accept the lot
Step 4 : If the number of defectives is greater than 1, reject the lot
Step 5 : If the number of defectives is equal to 1 and if a previous lot and succeeding lot resulted in acceptance, then accept the current lot otherwise reject it. The Maximum outgoing quality is 0.38%.

Conclusion:
In this article, Two sided - Complete Chain Sampling Plans CChSP(0,1) with average outgoing quality limit as an index is designed and developed. It is found that as the chaining index i increases the sample size converges to a constant. The complete Chain sampling plan gives more pressure on the producer if the quality deteriorates. These plans provide consumer an assurance regarding the outgoing quality or the quality of the lot after the inspection. Hence one can recommend this type of sampling plans for better quality control practice.

References: