# Inappropriate Strategies That Senior High School Students Use When Solving Routine Problems in Basic Algebra 

Patrick Kwabena Amoakoh<br>Tutor, Department of Mathematics and ICT, OLA College of Education, Cape Coast, Ghana


#### Abstract

The purpose of the study was to identify the inappropriate strategies that senor high school students used when solving routine problems in basic algebra. The study was conducted with second year students of Eguafo Senior High School, Edinaman Senior High School and Komenda Secondary Technical School; all situated in the Central Region of Ghana. The data used in the analysis were collected form 120 students of the various programmes in the Senior High School. The simple random sampling method was used in the selection of the sample and the descriptive sample survey was the research design used. Essay type achievement tests and interview were used in the data collection. The test involved 20 independent test items under 10 routine problems in basic algebra. The test items were carefully selected from various Senior High School Mathematics text books and WAEC past questions under the supervision of the supervisor.

Students were assembled in their classrooms for the administration of the tests. The tests were administered during official school hours. A total of thirty students, ten from each school were interviewed individually on their solutions to the test items. The failure strategies identified were observed to have comprised one or some combinations of a number of errors classified into six types as follows: a. Sign Errors, Misconception, Slip-shod Errors, Misapplication of RulesComputational Errors and Miscellaneous Errors. It was recommended that textbook authors in their text books should caution students on the possible common mistakes students make with each topic.


Key words: Junior High School, National council of Teachers of Mathematics, Static Sign Error

## Introduction

## Background of the study

Mathematics is a word that has been given meanings and varied interpretations widely from the time to time and from person to person. It is one of the oldest of all fields of study. He word mathematics which first meant that which was learnt (T Ivory, 1939.pp2) by the Greeks has now had many definitions and explanations.

For instance, Palling(1982) sees mathematics as been used in finding answers to questions and problems which arise in everyday life, trade and professions. A prominent English mathematician philosophy Berttrand Russel regards Mathematics as the subject in which we never know what we are talking about or whether what we are saying is true. Kline also propose that mathematics is the supreme and most remarkable example of the minds power to cope with problem $\backslash$ and as such it is worthy of study. (M. kline 1962 pp 7 ).

Bell (1980) stated that, it is generally accepted that being mathematically competent involves more than knowing a set of particular concepts, theorems and skills. It includes one's ability to use them in solving simple problems. However, it is possible to acquire a satisfactory knowledge on that particular concept, theorems and skills. Yet be totally incapable of solving the simplest problems. World problems can be considered as he bedrock of mathematics which deals with the relationships among symbolic forms.

However, the most simples and common ones among the mathematics definitions is the one given by Davis and Hersh (1963) that mathematics is a science of quantity (arithmetic and computations and space (geometric and questions concerning it spatial measurement. It can be seen that in this view era of modernization of the world there cannot be any meaningful development without knowledge of science and mathematics. Mathematics reveals hidden patterns that enables us to understand and the world around us as well as opens and facilitates logical and quantitative thinking abilities. Because of these reasons the education system of many countries including Gahna is seriously highlighting immensely on the development of mathematics study. For example in Ghana, ministry if education MOE (1998) see mathematics at the primary school level as very important and that much emphasis should be placed on knowledge, understanding and the skills to help the pupils develop the foundations of word problems in numeracy.

Hence, pupils must be able to competently, reason logically and solve word problem using concrete materials.

It is an undeniable fact from above examples that mathematics had still has many advantage and therefore its study is worthy it so them if it must be studied then by which medium must mathematics be instructed to pupils inorder to make its educational objectives be achieved. According to Bruner (1966) teaching is mostly facilitated by the medium of language and he answers the question. This implies that the language used in mathematics instruction for children is important thus if children cannot communicate in the language used during the lesson than their national language; children to school and are able to communicate effectively. Unlike most developing countries of which Ghana is no exception thee exist many local language spoken throughout he country aside the official language which is English. Mathematics is the core of all subjects, the backbone of all scientist advancement as well as the language of all scientists. Mathematics cures the vice of mental distraction and cultivates the habits of continues attention.

EulenberyanSubko(1968) considered word as a language for communicating certain kinds of ideas, as a logical structure of deducting reasoning and as a method of problems solving that target on the analyzing of relationships, solving verbal problems, evaluations and interpreting formulae.

## Statement of the problem

According to Chief Examiner Reports of West African Examinations Council (WAEC- 2012, 2013, 2014, 2015, 2016, and 2017) make it clear that the weakness of candidates' performance in core mathematics at the Senior Secondary School Certificates Examination (WASSCE) is mostly on test items from the content of word problems. These reports point out that most candidate have not conceived the concept of the use of brackets, the concepts of differences of two squares, the concept of change of subject, formulation of algebraic expressions, the concept of factorization and the use of negative and positive signs meaningfully.

Some studies have been conducted earlier on this problems in all attempts to identifying the failure strategies students use in solving outline problems in basic words problems (Prempeh 1998), Iddrisu (1999), Quianoo (2000), Agbozo and Asamoah (2002). Some of the errors identified were based on small samples and specific areas. For instance, Iddrisu (1999) carried on his study with 40 second year students' in Tamale which secondary school, Tamale which Agbozo and Asamoah (2002) carried out their study using second year student at Ghana secondary Technical school (GSTS) in Takoradi and 47 second year students from Archbishop Porters Girls Secondary school also in Takoradi and others. It is against this background that this research designed to identify some more failure strategies students consistently use in solving routine problems in basic algebra in mathematics in other school settings.

## Purpose / Objective of the study

The purpose of the study was to identify the inappropriate strategies that senior high school students used when solving routine problems in basic algebra.

## Research Questions

1. What are the inappropriate strategies that senior high school students used when solving routine problems in basic algebra?

## Materials/Methods

## Research Design and Rational For the Design

In this study descriptive research is used as the research design. This design has been adopted to base on the fact that it provides a descriptive picture of a situation that serve as a baseline data for consideration by the researcher and practitioner in making her discussion. The descriptive sample survey design involves collecting data in order to test hypothesis or to answer questions relating to situation of the study. The choice of the design is appropriate since the study is attempting to describe some aspect of the population.

## Population

The population consists of second year students of Eguafo Senior High School, Edinaman Senior High School and Komenda Secondary Technical School. A total of 120 students were drawn from the three schools.

## Sampling Technique

SHS two Students were selected for the study. This was due to the fact that the schools visited express unwillingness to release the form three students for the exercise. They say that they are in their examination period for which there was not any time to spare in extra activities. Besides, the test items were selected under the content of the first and second year of the syllabus. Thus, it is expected that by secondary form two almost all the topics in the items in senior secondary school mathematics syllabus might have been taught. To select the sample, the researcher used simple random sampling techniques to select intact classes from the following programs run by the school. Edinaman; General Art, Science, Business Home Economics and Visual Art. The names of the streams of the three programs were written on pieces of papers and put into a container and shuffled. One of the six streams selected at randomly was visual art, similar method was employed at Eguafo Senior High School to selects (a home economics class). Out of six of five programs-science, Business, General Art, Visual Art and Home economics. The same method was applied at Komenda Secondary Technical School. The same intact class was used because the research work was not purported to compare performance within the programs but only to identifying the difficulties students face in solving word problems in basic mathematics. In all 120 students were selected thus 40 from each school. The ages of boys range between 15 to 19 years, an average of 17 years while that of the girls range between 15 to 18 years with an average age of 16 years.

Table 1: Distribution of the number of students in the sample

| School stream | Swesco |  | Edinaman |  |  |  |  |  | Komenstech |  | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
|  | Boys | Girls | Boys | Girl | Boys | Girls |  |  |  |  |  |
|  | 6 | 4 | - | - | 8 | 4 | 22 |  |  |  |  |
| Agric science | 6 | 4 | 5 | 3 | 4 | 2 | 26 |  |  |  |  |
| Business | - | 6 | - | 4 | - | 6 | 16 |  |  |  |  |
| Home economics | 6 | 3 | 4 | 2 | 4 | 2 | 22 |  |  |  |  |
| General arts | - | - | - | - | 8 | 6 | 14 |  |  |  |  |
| Technical | 5 | 3 | 4 | 2 | 4 | 2 | 20 |  |  |  |  |
| Visual Arts | 25 | 20 | 13 | 11 | 28 | 22 | 120 |  |  |  |  |

The second year students were used for the study because the test items were selected under the contents of the first and second years of the senior high school mathematics syllabus. It was expected that by the second year most of the topics in the test have been learnt and therefore algebra is still fresh in the minds of the students.

Also at the time the research, second year students were readily available than the other students. Third year students were busily preparing for their final examination.

## Instruments

The main research instrument used for the data collection was an essay type achievement test based on basic algebra. Ten algebraic routine problems were carefully selected from various senior high school mathematics textbooks (1 and 2), WEAC past questions and WEAC chief examiners report all in accordance with the senior high school syllabus. For the sake of reliability and validity of the test, no attempts were made to modify the questions

## Scoring scheme or key

A partial credit award marking scheme was used in scoring the test. It contained solution(s) to each item used to detect the failure strategies used by the students. The scheme was drawn such that it provided alternative strategies (if any) to the routine problems

## Validity

To ensure face validity and content relevance, test items were selected under the supervision of the supervisor. The supervisor vetted the test items and based on his recommendation the test items were revised accordingly.

## Reliability

The reliability of the instrument was determined using the Crombach Coefficient Alpha (a). The coefficient was found to be 0.84

## Data Collection Procedure

The researcher visited Edinaman Senior High School, Eguafo Senior High School and Komenda Secondary Technical School. The researcher visited these schools with introductory letters from the Department of Mathematics \& ICT, OLA College of Education. Copies of the introductory letters were presented to the Assistant Heads (Academics) and the Heads of Mathematics Department of the schools. The researcher briefed them on the purpose and importance of the study. The tests were then conducted in the schools.

## Administration of the test

The administration of test in all schools followed the same procedure. Students were assembled in their classrooms in each school for the test. The tests were administered during the official school hours. Each student was presented with the same set of test items and answer sheets. Students were required to answer 20 independent test items under ten routine problems in basic algebra within 1 hour 30 minutes.

## Administration of the interview

A total of thirty students were interviewed. Students were briefed on the purpose of the interview in order to create a good rapport between the students and the researchers and also to motivate students to co-operate. The researcher interviewed the students individually and gave the students the opportunity to discuss the problems they encountered in the solving the test items. Finally, the papers were given back to students and they were asked to explain their solution process. The interview section lasted about forty-five minutes in each school.

## Theory/Calculations

## Description of Types of Errors

There were twenty test items for a sample of 120 students to be solved under ten routine problems in basic algebra. This, altogether the data contained 2400 written solutions. The search for a pattern for the written solutions led to classification of failure strategies. The failure strategies identified comprised errors or mistake categories into six types as shown in table 2.

Table 2: Types of failure strategies
Name of errors
types of errors

| Sign errors | I |
| :--- | :--- |
| Misconceptions | II |
| Slip-shod errors | III |
| Misapplication of rules | IV |
| Computational Errors | V |
| Miscellaneous Errors | VI |

Source: Field data, 2017
The researchers observed that the six types of errors described about $95 \%$ of the 1800 responses used by students in responding to the test item. The remaining were too vague to classify. It was found that about $87 \%$ of the cases can be described as single type of errors. The errors are described with illustrative example.

## Sign Errors (TypeI)

A failure strategy was classified as a sign error when students fail to change the operation sign of a term when transposing it across the equal sign or when students fail to recognize an operation sign as being [part of a term when collecting and regrouping like terms in algebraic expressions.

Sign errors may be caused by the loss of parenthesis or by the belief that a minus sign is a negative number. For example in an attempt to simplify the expression in item 1i. That is:
$X\left(x-2 \_-5 y(3 x-6 y)\right.$
Some student proceeded as follows:
$X(x-2 y)-5 y(3 x-6 y)=x^{2}-2 x y-15 x y+30 y^{2}$
$=x^{2}-2 x y-15 x y-30 y^{2} \ldots \ldots \ldots \ldots \ldots \ldots$ (Type I)
$=x^{2}-17 x y-30 y^{2}$
During the interview session, a student confidently said that "I maintained the signs because no term crossed the equal sign". He thought the sign of a term changes only when it crosses the equal sign.

Other examples were observed in the following items. These were the solutions presented by some students.

$$
\begin{gathered}
\frac{1}{4}(3 x-5)=2(x-1)-3 \\
(3 \mathrm{x}-5)=8(\mathrm{x}-1)-3 \\
3 \mathrm{x}-5=8 \mathrm{x}-12 \\
3 \mathrm{x}+8 \mathrm{x}=-8-12-5 \ldots \ldots \ldots \ldots \ldots \\
11 \mathrm{x}=-25 \\
\mathrm{X}=-\frac{25}{11} \\
\left\{\mathrm{x}: \mathrm{x} \in \mathrm{R}=-\frac{25}{11}\right\} \\
\frac{2 x-1}{3}-\frac{x-2}{4}=1
\end{gathered}
$$

$$
\begin{gathered}
4(2 x-1)+3(-x-2)=12 \ldots \ldots \ldots \ldots \ldots \text { (TypeI) } \\
8 x-3-3 x-6=12 \\
8 x-3 x=12+4+6 \\
5 x=22 \\
X=\frac{22}{5} \\
\left\{x: x \in R=\frac{22}{5}\right\}
\end{gathered}
$$

## Misconception (Type II)

This type of error is committed when students clearly show the misunderstanding of a concept in their strategy. However, students may show some level of consistency. These errors are mostly made when students change the given surface structure of an expression into a simple form to suit their understanding. Most of the examples were observed when students mistook expression as equation.

In simplifying he expression in item lii, some students proceeded by multiplying the lowest common multiple of he denominators through the expression thereby cancelling out the fraction as follows:
$\frac{3 x-y}{4}-\frac{4 x-6}{3}$
$12 \frac{(3 x-2)}{4}-12 \frac{(4 x-6)}{3} \ldots \ldots \ldots \ldots \ldots . . . . . . . . .$.
$3(3 x-y)-4(4 x-6)$
When some students were interviewed as to how they obtained these results, one student said "it is better to clear the fraction and work in a straight line.

## Slip-Shod Error (Type III)

Slip - shod error occurred when a student knew exactly what to do, how to do it and was actually doing it but went astray during the process. Here, the student was able to identify the correct strategy for solving the problem.

Some slip-shod errors are mainly caused by carelessness or sheer laziness to use bracket. Some students had very bad handwriting that caused them to misread their own writing hence they ended up with wrong answers.

Examples were observed in items that involved the use of and application of difference of two squares.
Most students displayed their misapplication of the difference of two squares as follows:

$$
\begin{aligned}
& \text { liv. } \quad \frac{49 u^{2}-16 v^{2}}{5 u v} \frac{v^{2} u^{2}}{7 u-4 v} \\
& \quad \frac{49 u^{2} v-16 v^{3} u}{35 u-20 v} \\
& \begin{aligned}
49 u^{2}-16 v^{3} u \\
35 u-20 v
\end{aligned} \\
& \text { 2ii } \quad(\mathrm{x}-\mathrm{a})(3 \mathrm{x}-2 \mathrm{a})-(\mathrm{x}-\mathrm{a})^{2} \\
& \quad \\
& \\
&
\end{aligned}
$$

$$
\begin{aligned}
& 3 x^{2}-2 a x-=3 a x+2 a^{2}-x 2+2 a x-a^{2} \\
& a^{2}-2 a x-3 a x+2 a x+3 x^{2}-x^{2}
\end{aligned}
$$

## Misapplication of rules (Type iv)

Another example of the error in applying rule is seen in test item 1iv.

$$
\begin{aligned}
& \frac{49 u 2-16 v 2}{5 u v} \frac{\left(v^{2} u^{2}\right)}{7 u-4 v} \\
& \frac{49 u^{2}-16 v 2}{5 u v} \frac{v^{2} u^{2}}{7 u-4 v}
\end{aligned}
$$

$\frac{49 u 4 v^{2}-16 u^{2} v 4}{35 u^{2} v-20 u v^{2}}$
$\frac{7 u^{2} v-4 u v^{2}}{5-6}$. (Type iv)
$-\left(7 u^{2} v-4 u v^{2}\right)$
$4 u v^{2}-7 u^{2} v$
$u v(4 v-7 u)$
Here, the student misapplied the cancellation rule. He did not treat the terms in eh numerator as an entity or the terms in the denominator but cancelled out in isolation.

The use of the quadratic formula:
$X=-b+b 2-4 a c v$
2a
Of the quadratic equation of the form $a x^{2}+b x-x=0$ was also misapplied by students. Some students quoted the quadratic formula wrongly whilst others were unable to identify the values of he coefficients $\mathrm{a}, \mathrm{b}$ and c correctly.

For instance in solving the quadratic equation

$$
3 n^{2}-8 n-2=0
$$

In test item 5(ii)., some students presented the following:

$$
\begin{aligned}
& 3 n^{2}-8 n-2=0 \\
& a=3, b=8, c=2 \ldots \ldots \ldots \ldots \ldots \text { (type iv) } \\
& n=\frac{-8+\sqrt{ } 8^{2}-4(3)(2)}{2(3)} \\
& n=\frac{-8+\sqrt{ } 64-24}{6} \\
& n=-\underline{8+\sqrt{ } 40} \\
& 6
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{n}=-4 \pm 40 \\
& \mathrm{n}=-\frac{4+\sqrt{ } 10}{3} \underline{3} \\
& \mathrm{n}=-\underline{4}+\sqrt{10} \text { or } \mathrm{n}=-4=\sqrt{ } 10 \\
& 3333
\end{aligned}
$$

The respondent demonstrated a clear misunderstanding of the rule of changing an inequality sign when an inequality is divided or multiplied through by a negative quantity. Test item for shows the misapplication of this rule.

$$
\begin{aligned}
& \frac{1}{2}(3 x-5) \leq 3 x-12 \\
& 3 x-5 \leq 293 x+12) \\
& 3 x-5 \leq 6 x+24 \\
& 3 x-6 x<29 \\
& x \leq-29 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . .(\text { Type iv) } \\
& 3 \\
& \{X: X \in R, X=5\}
\end{aligned}
$$

Distribution errors were also committed students did improper distributions. In an attempt to factorize completely the expression

$$
\begin{aligned}
& (x-a)(3 x-2 a)-(x a)^{2} \\
& x(3 s-2)-a(3 x-2 a)-\left(x^{2}-a^{2}\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . .(\text { Type eiv }) \\
& 3 x^{2}-2 a-3 a x+2 a^{2}-x^{2}+a^{2} \\
& 3 a^{2}-5 a x+2 x^{2}
\end{aligned}
$$

Another misapplication of rule was identified when students solved quadratic equation in test item 5.

```
\(5 x^{2}-8 x+3=0\)
\(5 x^{2}-8 x=-3\)
\(\mathrm{X}(5 \mathrm{x}-8)=-3\)
\(\mathrm{X}=-3\) or \(5 \mathrm{x}-8=-3\)
```

$\qquad$

```
\(X=-3\) or \(5 x=5\)
\(X=-3\) or \(x=1\)
```

Here, students wrongly extended the fact that if
$\mathrm{A}(\mathrm{x}-\mathrm{b})=0$, then $\mathrm{a}=0$ or $\mathrm{x}-\mathrm{b}=0$ (ie. If $\mathrm{ab}=0$, then $\mathrm{a}=0, \mathrm{~b}=0$ or $\mathrm{a}=0, \mathrm{~b}=0$
To other real numbers

## Computation errors(V)

They are errors made during computation. Here, students are able to identify what to do and how to solve a problem but make calculation mistakes. These errors may be due to sheer carelessness. Test item 3 i exhibits a computational error. In finding the truth set of

$$
\begin{gathered}
1(3 x-5)=2(x-1)-3, \\
4
\end{gathered}
$$

Students presented the following

$$
\begin{aligned}
& \underline{1}(3 \mathrm{x}-5)=2(\mathrm{x}-1)-3 \\
& 3 \mathrm{x}-5=8 \mathrm{x}-8-12 \\
& 3 x-8 x=-8-12+5
\end{aligned}
$$

$$
X=5
$$

$$
\{\mathrm{X}: \mathrm{X} \in \mathrm{R}, \mathrm{X}=5\}
$$

## Miscellaneous errors (VI)

Miscellaneous errors are other that were discovered by researches but could not be classified under any of the other failure strategies. Some of these miscellaneous errors are discussed
a. Students' inability to determine the final stage of a factorization. Hence, stop prematurely or go ahead to begin expansion.
In item 1iii, where students were asked to simplify the expression:

$$
\begin{aligned}
& \mathrm{d}^{2}-2 \mathrm{ad}-2 \mathrm{~d}^{2}+8 \mathrm{ad}+3 \mathrm{~d}^{2}-20 \mathrm{ad} \\
& \mathrm{~d}^{2}-2 \mathrm{~d}^{2}=3 \mathrm{~d}^{2}-2 \mathrm{ad}+8 \mathrm{ad}-20 \mathrm{ad} \\
& 2 \mathrm{~d}^{2}-14 \mathrm{ad} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{aligned}
$$

Most students ended as shown above.
They were expected to proceed as follows;

$$
2 \mathrm{~d}(\mathrm{~d}-7 \mathrm{a})
$$

Another example can be observed in test item 2ii as follows:

$$
\begin{aligned}
& (x-a)(3 x-2 a)-(x-a)^{2} \\
& (x-a)[(3 x-2 a)-(x-a)] \\
& (x-a)(2 x-a)
\end{aligned}
$$

Students were expected to end as shown above
They rather ended or proceeded as follows:

$$
\begin{aligned}
& x(2 x-a)-a(2 x-a) \\
& 2 x 2-a x-2 a x+a^{2} \\
& 2 x 2-3 a x+a^{2}
\end{aligned}
$$

b. Some student copied out wrong questions different from the question stated on question paper.

Others did not read directions clearly. Some students found the truth set of
Instead of $\underline{1}(3 x-5) \geq 3 x+12$
2
In item 4i. $\underline{1}(3 x-5) \leq 3 x+12$
2
c. Student use and loss of brackets caused may errors in their solution process. Most of these errors were observed in item 7 where students given
$\mathrm{a}=2, \mathrm{~b}=-3$; and $\mathrm{c}=-2$
Evaluate

$$
\frac{a b^{2}-c^{2}}{2 b c}+\frac{a^{2}}{2 b+c}
$$

The following are some of the substitutions presented.
i $\frac{(2)(-3)^{2}}{2(-3)(-3)}+2(\overline{20+(-2)}$
$2-3^{2}-2^{2} \quad 2^{2}$
ii $\underline{2-3^{2}-2^{2}}+2^{2}$

$$
2(-3)(-2) \quad 2(2)+(-2)
$$

d. Students had a lot difficulty understanding the story problem
e. Students used circuitous, longer and difficult methods that were prone to errors in solving problems.

## Findings of the study

The failure strategies used by students and their corresponding percentages are recorded in Table 3.
Table 3: Distribution of the number of times students make specific kinds of errors

| Type of error | number of times <br> Students made the error | percentage of times <br> students made the errors a (\%) |
| :--- | :---: | :---: |
| Sign errors | 76 | 27.3 |
| Misconceptions | 38 | 13.6 |
| Ship-shod | 30 | 10.7 |
| Misapplication of rules | 54 | 19.4 |
| Computational error | 20 | 7.1 |
| Miscellaneous errors | 60 | 21.5 |
| Total | 278 | 100 |

Source: Field data, 2017

## The major findings include the following:

1. Sign errors were the most common failure strategies identified. It was observed that about $63 \%$ of the students made sign errors in at least one item. This indicates that most senior high students have problems with changing the sign of terms when transposing and the sign of terms involving brackets.
2. About $45 \%$ and $30 \%$ of students made conceptual errors and misapplied rules respectively. This result indicates that concepts are not well formed by students during the instructional periods. Most student treated equations as expressions and vice versa.
3. The slip-shod errors and computational errors students made can be avoided if students read over and check their solution process.
4. Some test items exhibited more than failure strategy. The following solutions that students provided illustrate multiple failure strategies.
In item 1ii, as student made a


Misconception error by solving the expression as though it was equation. The students proceeded to make a slip shod errors as well as sign error.

In simplifying the expression, the student worked as follows,

$$
\begin{aligned}
& \frac{3 x-y}{4} \quad \frac{4 x-6}{3} \\
& \frac{12(3 x-y)}{3}-\underline{12(4 x-6)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text { (Type ii) } \\
& 4 \text { 3 } \\
& \text { 12(3x-y) - 12(4x-6)........................................... (Type iii) } \\
& 36 x-y-48 x-72 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .(T y p e ~ i) \\
& -72-12 x-y
\end{aligned}
$$

An example of multiple failure strategy was observed in item 2iii where students were to factorize completely the expression.

$$
(x-1)(x+2)^{2}-(x-3)^{2}(x-1)
$$

$$
\begin{aligned}
& (x-1)\left[(x+2)^{2}-(x-3)^{2}\right] \\
& \left.(x-1)\left[x^{2}+4 x+4\right)-\left(x^{2}-6 x+9\right)\right] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \text {. (Type iv) }
\end{aligned}
$$

Here, the student first committed a misapplication of rule error by the student's inability to identify and use the difference of two squares.

A slip shod error was committed when student wrote

$$
x^{2}+4 x-4
$$

Instead of

$$
x^{2}+4 x+4
$$

The student did not know at what stage the factorization must end, did premature factorization and hence committed a miscellaneous error.

In an attempt to evaluate the expression

$$
\frac{a b^{2}-c^{2}}{2 b c}+\frac{a^{2}}{2 b+c} \text { given }
$$

$\mathrm{A}=2, \mathrm{~b}=-3$; and $\mathrm{c}=-2$, some students made sign errors, slipshod errors as well as computational errors. The following is what a student presented.

$$
\begin{gathered}
\frac{a b^{2}-c^{2}}{2 b c}+\frac{a^{2}}{2 b+c} \\
\frac{(2)-3^{2}-3^{2}}{2(-3)(-2)}+\frac{2^{2}}{2(-3)+(-2)} \quad \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~(T y p e ~ i) ~
\end{gathered}
$$

## Discussion of the funding

According to Prempeh (1998) identified four types of failure strategies namely; (i) Errors in reasoning, (ii) Errors in computation (iv) Errors in applying rules and (v) Miscellaneous errors as compared to the six types identified by the researcher. Errors in computation and errors in applying rules are identified by the researcher.

Errors in computation and errors in applying as identified by Prempeh (1998) are consistent with the computational errors and misapplication of rules respectively identified by the researcher. The researchers found two other failure strategies as slipshod errors and miscellaneous errors.

Some researchers (Iddrisu, 1998, Balaara et al, 2007) classified failure strategies into six namely (a) errors in reasoning (b) static sign errors, (c) errors in applying rules (d) misconceptions, (e) slip shod and (f) miscellaneous errors; whilst Farouk and Wea (2005) also found six categories of failure strategies namely (a) errors in reasoning (b) errors in computation, (c) misapplication of rules (d) static sign errors, (e) misconceptions, and (f) miscellaneous errors

The researcher classification of failure strategies as misconceptions contradicts that of the other researchers (Idrissu, 1998; Farouk and Wea, 2005; Balaara et al, 2007). The researcher identified misconceptions as a
failure strategy. This comprised the failure strategies the other researchers categorized as errors in reasoning and misconceptions.

The researcher's use of sign errors was consistent with static sign errors of the other researchers. The researcher observed that $2.8 \%$ of the errors students committed were sign errors and hence sign errors were the most common failure (Farouk and Wea, 2005 Balaara et al, 2007). Farouk and Wea (2005) observed that static sign errors were $80 \%$ of the errors committed by students whist Balaara et al (2007) found that static sign errors consisted $44.2 \%$ of the errors students committed.

## Conclusion

The purpose of the study was to identify the inappropriate strategies that senior high school students used when solving routine problems in basic algebra. It can be recommended that the failure strategies identified were observed to have comprised one or some combinations of a number of errors classified into six types as follows: a. Sign Errors, Misconception, Slip-shod Errors, Misapplication of Rules, Computational Errors and Miscellaneous Errors.

## Recommendations/Suggestions

The following recommendations/suggestions are made to mathematics teachers, curriculum planners, test book authors, tutors in teacher training institutions, examiners as well as students based on the results and conclusions reached in thisresearch.

- Mathematics educator in teacher training institutions should explain to teacher trainees how and why these failure strategies occur.
- Textbook authors in their text books should caution students on the possible common mistakes students make with each topic.
- Curriculum developers and planners should include more work on algebra in the curriculum because algebra is the bedrock of mathematics; take it away and nothing is left of mathematics.
- Examiners should use failure strategies to obtain attractive distracters for multiple choice questions. The failure strategies students' use in solving routine problems is basic algebra can serve as a way for obtaining plausible attractive distracters for multiple choice test items.
- It is therefore recommended that further research be conducted on a larger scale to cover as many senior high school students as possible in the country.
- Also, further research should be conducted on the failure strategies in other areas of mathematics like Geometry and Trigonometry at the senior high school.
- Students should be encouraged to study their solution process after obtaining them. They should read over and check their work to avoid non conceptual errors. Student must make it a requirement to read over and check the solution process of each problem they solve.
Students must practice more since practice makes man perfect.
- Teachers should effectively teach skills that will equip students to determine and use the appropriate strategies in solving specific routine problems.
- Teachers should discuss and caution students during instructional period on the common mistakes involved in topics. Students will then be conscious of such failure strategies and more likely to avoid them.
- Teachers should carefully explain algebraic conceptions with examples, non-examples and counter examples.
- In-service training, refresher courses and workshops should be organized regularly for mathematics teacher in order to update their knowledge about modern trends in Mathematics and Mathematics Education.


## References

[1] Agbozo, K.\&Asamoah, S (2002). Failure Strategies in Solving Routine Problem in Basic Algebra in the Senior Secondary School (UnpublishedProject).University of Cape Coast.
[2] Balaara, E.E.,Logo, G.P., \&Sappor, W.L. (2007) Failure Strategies in Solving Routine problem in Basic Algebra in the Senior Secondary School (Unpublished Project). University of cape coast.
[3] Barnett, M., \&Kearns., (1971) Intermediate Algebra: Structure and Use. (5ed.) New York: McGraw Hill.
[4] Bell, E.W. et al (1980) Research on Learning and Teaching A Review of Research in Mathematics Education Part A. London: Chipeham, Anthony Rowe Limited.
[5] Booth, L. (1984) Algebra: Children's Strategies and Errors. A report of the Strategies and Errors in Secondary Mathematics Project. Windsor: Nfernelson.
[6] Churchman, R.E., (1981). Handbook of Research on Teaching (3ed.) New York: McGraw-Hill.
[7] Dantzig.T. (1947).Number, the Language of Science: A Critical Survey written for the Culture Nonmathematician. London: Allan and Uzwin.
[8] Farouk, A.O., Wea, E..(2005).Failure Strategies in Solving Routine Problem in Basic Algebra in the Senior Secondary School (Unpublished Project).University of Cape Coast.
[9] Fawcett, H.P (1970), The Teaching of Mathematics from Counting to Calculus, Ohio: Charles. E Merrill Publishing Co.
[10] Frankland, L. (1960). The Language of Mathematics.Norwick, London: John Murrary
[11] Gray, E.M., \& Tall, D.O., (1991). Duality, Ambiguity and Flexibility in Successful Mathematical Thinking. Assisi, Italy.
[12] Hunt.E B (1962).Concept learning on Information Process Problems. New York: Welly.
[13] http://en.wikipedia.org/
[14] Hart, K.M (Ed) (1981). Children Understanding of Mathematics. London: John Murray Publishers Limited.
[15] Iddrisu, M.G. (1999). Failure Strategies in Solving Routine Problems in Basic Algebra in The Senior Secondary School (Unpublished Project). University of Cape Coast.
[16] Kline, M. (1972) Mathematical Thought from Ancient to Modern Times. New York: Oxford University Press.
[17] Kline, M. (1985).Mathematics for the Non-Mathematician. New York: Dover Publication Inc.
[18] Krygowska, Z. (1957). On Dangers of Formalism in Teaching Algebra in the SchoolArchimade, Toronto: John C, Winston. Co Limited.
[19] Kuchemann, D. (1981). Children's Understanding of Mathematics Murrary: London.
[20] MacGregor, M., \&Staccey, K. (1997).Understanding Algebraic Notation, Educational Studies in Mathematics.Vol 33, Number, June 1997, 1-19.
[21] Peirce, B. (1881) Linear Associative Algebra. American Journal of Mathematics (Vol. 4, No. 1/4).
[22] Prempeh.E.A, (1998).Failure Strategies in Solving Routine Problems in Basic Algebra in Senior Secondary School(Unpublished Project) University of Cape Coast.
[23] Quainoo, J. (2000). Failure Strategies in Solving Routine Problems in Basic Algebra in Senior Secondary School (Unpublished Project).University of Cape Coast.
[24] Radataz, H. (1979). Error Analysis in Mathematics Education.Journal for Research in
[25] Mathematics Education, Vol. 10, No. 3, 163 - 172.
[26] Senk, S.L. et al (1966). School of Mathematics project, University of Chicago (2ed).Chicago: Scott Foreman.
[27] Smith, K.J. (1987). The Nature of Mathematics (5ed) California: Brooks \& Cole Publishing Company,
[28] Snader, D.W. (1954). Algebra: Meaning and Mastery. Toronto: John C. Winston Co. Limited.
[29] Steen, L.A. (1992) School Science and Mathematics.Official Journal of School Science and Mathematics. Vol. 22, Number 2.
[30] Stein, E.I (1956).Algebra in Easy Steps. London: D Van Nostrand Company Inc. Pp 3-4
[31] Swokowski, E.W. (1986) Fundamental Algebra and Trigonometry. Boston: Prinde, Weber and Schmidt.
[32] Steinberg. (1991). The Algebra ofequivalent Fractions.International Journal of Mathematical Education in Science and Technology (IJMEST) Vol. 18, No. 41998.
[33] Willougby, S.S. (1967) Contemporary Teaching of Secondary Mathematics New York: John Wiley and Sons Inc.

